10 Triangles

10.0 Triangle Concepts

In this chapter, we will:

- Introduce and define the triangle.
- Introduce the concept of triangulation of polygons.
- Examine the sum of the interior angles of the triangle.
- Examine the exterior angles of the triangle.
- Define remote angles and investigate their properties.
- Introduce the concept of classifying triangles.
- Describe the classification of triangles by the length of their sides including equilateral, isosceles, and scalene triangles.
- Describe the classification of triangles by their interior angles including acute, right, obtuse, and equiangular triangles.
- Investigate the nomenclature for parts of triangles.
- Define and investigate the medians, angle bisectors, bases, and altitudes of triangles.
- Investigate the intersection of the altitudes in triangles.

10.1 Introduction to Triangles

In this chapter, we will begin the investigation of triangles. Triangles may be the most important geometric objects in geometry, and we will spend a great deal of time and effort on them. This chapter is the introduction to triangles. Be certain that the student understands these introductory concepts before moving on:

- A triangle has three sides, three vertices, and three interior angles.

- The written symbol for a triangle is $\triangle$. The most common way to name a triangle is to name it by it's three interior angles. The name of the triangle with interior angles X, Y, and Z is $\triangle XYZ$.

Topic Questions

Describe the geometric object $\triangle PQR$. (The object is a triangle with vertex points P, Q, and R. its interior angles are $\angle P$, $\angle Q$, and $\angle R$. It has three interior angles and three sides.)

10.2 Triangulation

Complex geometric objects can be broken down into triangles. This process is called **triangulation**.

Topic Questions

What is triangulation? (The process of dividing complex geometric objects or polygons into triangles.)

What are the limitations of triangulation? (The object being triangulated must have straight sides.)

10.3 Sum of the Interior Angles of a Triangle

The sum of the measures of the interior angles of a triangle is 180°. This fact can be used to find unknown angles when limited information is given about triangles.

Topic Questions

Given $\triangle ABC$ (triangle ABC), if the measure of angle $\angle A$ is 50° and the measure of $\angle B$ is 30°, what is the measure of $\angle C$? (Since the sum of the measures of $\angle A$, $\angle B$ and $\angle C$ equals 180°, the measure of $\angle C$ is: $180^\circ - 50^\circ - 30^\circ = 100^\circ$.)

10.4 Exterior Angles of a Triangle

An exterior angle in a triangle is the adjacent angle of an interior angle.

The exterior angle is formed by extending one of the sides of the triangle. The interior angle and exterior angle are supplementary, that means the sum of their measures is 180°.

Topic Questions

$\angle A$ is the interior angle of a triangle and $\angle B$ is the exterior angle. If the measure of $\angle A$ is 80°, what is the measure of $\angle B$? (These two angles are supplementary, that means their sum is 180°. Therefore, the measure of angle B is: $180^\circ - 80^\circ = 100^\circ$.)

10.5 Remote or Nonadjacent Angles

A remote or nonadjacent angle is an angle that does not share a common side with the angle under discussion.

Since the sum of the measures of the three interior angles of a triangle is 180°, and the exterior angle is supplementary to its adjacent interior angle, we know that the following must be true:

- **The measure of an exterior angle of a triangle is equal to the sum of the measure of the two nonadjacent interior angles.**

Topic Questions

In $\triangle ABC$, if $\angle A$ is an interior angle of a triangle, and $\angle Q$ is the exterior angle with a measure of 100°, what is the sum of the measures of $\angle B$ and $\angle C$? (By the earlier theorem, we know that the sum of the measures of the remote angles $\angle B$ and $\angle C$ must be equal to the measure of the exterior angle $\angle Q$. The sum of the measures of $\angle B$ and $\angle C$ is therefore equal to the measure of $\angle Q$: 100°).

10.6 Introduction to Classifying Triangles

In this sub-chapter, we introduce the classification of triangles by side and angle properties.

Topic Questions

Name two ways in which triangles are classified? (Triangles are classified by the properties of their sides and their interior angles.)

10.7 Classifications of Triangles by Sides

The classification of triangles by sides is determined by the number of congruent sides.

10.8 Equilateral Triangle

Triangles in which all three sides have the same length are called **equilateral** triangles.

Topic Questions

Triangle $\triangle PQR$ has three sides that all have the length 10. What kind of triangle is it? (Since all three sides have the same length, it is an equilateral triangle.)

10.9 Isosceles Triangle

Triangles that have at least two sides of the same length are called **isosceles** triangles.

Topic Questions

Triangle $\triangle JKL$ has two sides with the same length. What kind of triangle is it? (Since it has two sides of the same length, it is an isosceles triangle.)

10.10 Scalene Triangle

A triangle in which all three sides are a different length is called a **scalene** triangle.

Topic Questions

Triangle $\triangle XYZ$ has sides that are all of different length. What kind of triangle is it? (Since all of its sides are different length, it is a scalene triangle.)

10.11 Summary of Classification of Triangles by Sides

Number of equal sides:	Name of triangle:
1	scalene triangle
2	isosceles triangle
3	equilateral triangle

10.12 Classification of Triangles by Angles

The classification of triangles by angle is determined by whether or not the triangle has angles less than, equal to, or greater than 90°. Triangles are also classified by the number of congruent angles they contain.

Topic Questions

Name two ways in which triangles are classified by angle. (Triangles can be classified by whether or not they have angles less than 90°, equal to 90°, or greater than 90°. Triangles can also be classified by the number of congruent angles they have.)

10.13 Acute Triangles

A triangle where all of the interior angles are acute (that is, less than 90°) is an **acute** triangle.

Topic Questions

All the angles in $\triangle ABC$ are less than 90°. What kind of triangle is it? (acute triangle.)

10.14 Right Triangles

A triangle that contains a right angle (that is, an angle of 90°) is called a **right** triangle.

Sometimes right triangles are written using a special symbol, $\llcorner$, rather than the normal symbol for triangle $\triangle$. It is a picture of a small right triangle. So, the right triangle with vertex angles A, B, and C is written: $\llcorner ABC$. The student must be careful with this because this symbol is not always used with right triangles.

Topic Questions

What kind of triangle is $\llcorner JKL$? (Because it contains the right triangle symbol it is a right triangle.)

In $\triangle QAZ$ the angle $\angle Q$ has the measure of 90°. What kind of triangle is it? (Because this triangle contains an angle of 90° is a right triangle. Notice that the symbol for a right triangle was not used.)

10.15 Obtuse Triangles

A triangle in which *one* of the interior angles is larger than 90° is an **obtuse** triangle. A triangle can only contain one angle larger than 90°.

Topic Questions

In $\triangle JKL$, the angle $\angle K$ has the measure of 100°. What kind of triangle is it? (Because this triangle contains an angle larger than 90°, it is an obtuse triangle.)

10.16 Equiangular Triangles

A triangle having all angles of equal measure is an equiangular triangle.

Topic Questions

Given $\triangle ABC$ where the measures of $\angle A$, $\angle B$, and $\angle C$ are all equal, what kind of triangle is it? (Because the measures of all the angles are the same, it is an equiangular triangle.)

What are the measures of the angles $\angle A$, $\angle B$, and $\angle C$? (Since we know that the sum of the measures of the three angles must be equal to 180°, and must be the same, then they are each equal to 60°.)

10.17 Summary of the Triangle Family
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10.18 Names for the Sides and Angles of Triangles

In an isosceles triangle, the two sides of equal length are called legs. The third side is called the base.

In an isosceles triangle, the two equal sides form the vertex angle.

In a right triangle, the side opposite the right angle is called the hypotenuse.

The other two sides in a right triangle are called the legs.

10.19 Medians, Angle Bisectors, and Altitudes of Triangles

Properties and line segments within a triangle have special names. These names include medians, angle bisectors, and altitudes.

10.20 Bases of Triangles

Any side of a triangle can be called its base.

10.21 Altitudes of Triangles

The altitude of a triangle is the shortest distance from the line containing the baseof the triangle to the vertex point that is opposite the base. The shortest distance from any point to a line will be perpendicular to that line. The altitude might not be inside the triangle.

Every triangle has three bases and three altitudes.

Topic Questions

What is an altitude of a triangle? (The shortest distance from the line containing the base of the triangle to the vertex point opposite the base.)

Is an altitude always inside the triangle? (No, the altitude may be outside the triangle.)

10.22 Intersection of Altitudes in Triangles

If you draw all three altitudes in any triangle, you will find the three altitudes intersect at a single point.

In an acute triangle, the three altitude lines intersect inside the triangle.

In a right triangle, the three altitude lines intersect at the vertex containing the right triangle.

In an obtuse triangle, the three altitude lines intersect outside the triangle.

Topic Questions

Do the altitudes in a triangle intersect? (Yes, they intersect at the single point.)

Where do they intersect? (It depends on the type of triangle. In an acute triangle, they intersect inside the triangle; in a right triangle, they intersect at the vertex containing the right angle; and in an obtuse triangle, they intersect outside the triangle.)

10.23 Angle Bisectors of Vertex Angles of Triangles

An angle bisector is a line segment that bisects a vertex angle.

A triangle has three vertex angles; therefore, every triangle has three angle bisectors.

Topic Questions

How many angle bisectors does a triangle have? (Three: one for each vertex angle.)

10.24 Medians of Triangles

The median of a triangle is the line segment with one endpoint on the vertex of an angle and the other endpoint on the midpoint of the side which is opposite to that same angle.

All triangles have three medians, one median for each vertex angle.

The three medians intersect at a single point inside the triangle.

Topic Questions

How many medians does a triangle have? (Three:one for each vertex angle.)

Where do these medians intersect? (The three medians always intersect on the inside of the triangle no, matter what type of triangle it is.)

Student Workbook Problems with Answers

10 Triangles

10.1 Introduction to Triangles

1. The triangle has **three** sides, **three** vertices, and **three** interior angles.

2. The length of the side of the triangle can be any number.

   No, the length of every side must be greater than zero.

3. The sides of the triangle may be curved.

   True **False (straight line segments)**

4. The triangle is composed of lines.

   True **False (line segments)**

5. The vertex points of the triangle are in different planes.

   True **False (they must be in the same plane)**

6. The written symbol for triangle is

   A. T
   B. >
   C. ⊾
   D. △

7. What is △AVR?

   A triangle with its vertices at points A, V, and R.

8. What is wrong with △ABCD?

   It cannot be a triangle because it has too many vertices.

10.2 Triangulation

1. Triangulation is the process of dividing more complex geometric objects into triangles.

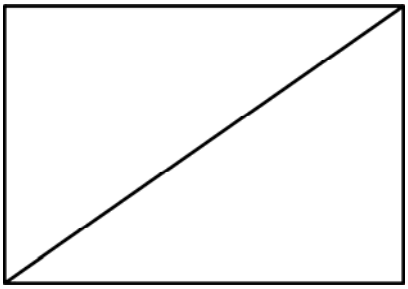
   True False

2. You can triangulate any geometric object.

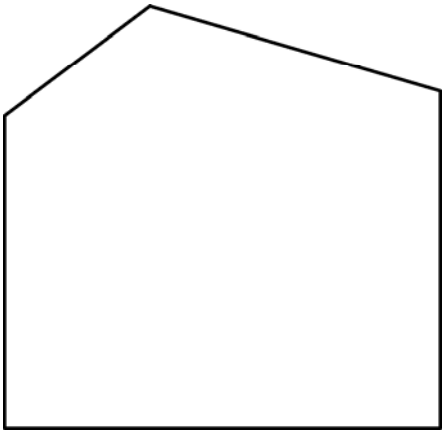
   True **False (the object must have straight sides)**

3. Triangulate this object.

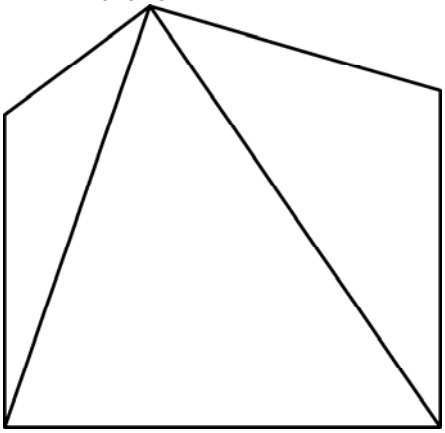
answer:



4. Triangulate this object.



answer:



10.3 Sum of the Interior Angles of a Triangle

1. The sum of the measures of the **interior** angles of the triangle is **180°**.

2. Given △ABC where $m\angle A = 90^\circ$ and $m\angle B = 60^\circ$, what is $m\angle C$? **30°**

3. Given △XYZ where $m\angle A = 80^\circ$ and $m\angle B = 80^\circ$, what is $m\angle C$? **20°**

4. A given geometric figure has two interior angles that are right angles. Can it be a triangle?

   No. 90° + 90° = 180°. There is not enough interior angle left to make a third angle.

5. A given geometric figure has interior angles that are all congruent. Can the measure of these angles be less than 90°.

   Yes. An equilateral triangle has 60° interior angles.

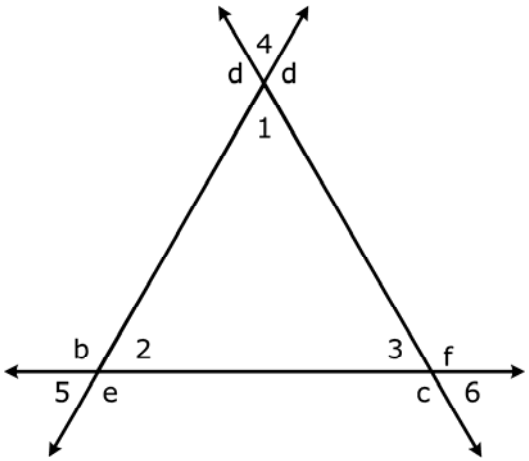
6. Is it possible for a triangle to have interior angles that are in the ratio 1 to 2 to 3?

   Yes. 30°, 60°, 90°

10.4 Exterior Angles of a Triangle

1. The exterior angles of a triangle are **supplementary** to the interior angles.

   For questions 2-10, use the drawing below:



2. Which angles are congruent to ∠1? Why? **∠4. Vertical angle.**

3. Which angles have equal measure to ∠2? Why? **∠5. Vertical angle.**

4. Which angles are supplementary to ∠3? **∠f, ∠c**

5. If $m\angle 2 = 60^\circ$, what is $m\angle 5$? **60°**

6. If $m\angle 2 = 60^\circ$, what is $m\angle b$? **120°**

7. If $\angle 1 \cong \angle 2 \cong \angle 3$, what is $m\angle e$? **120°**

8. Can $m\angle f = 180^\circ$
No. The interior angle would be zero.

9. If $\angle b \cong \angle 2$, what is true about $\angle 1$ and $\angle 3$?
Both angles must be smaller than 90°

10. Can $m\angle d = m\angle 5$?
No.

10.5 Remote or Nonadjacent Angles

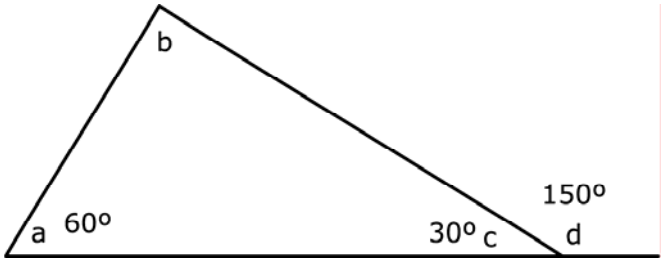
1. A remote angle is a nonadjacent angle.

True False

2. In triangle $\triangle ABC$, if $m\angle A = 30^\circ$ what is sum of the measure of the remote angles? What are the remote angles?

150°. Angles ∠B and ∠C are the remote angles of ∠A.

3. If $m\angle a = 60^\circ$ and $m\angle d = 150^\circ$, what is $m\angle b$? **90°**



10.6 Introduction to Classifying Triangles

1. Triangles can be classified by the properties of their angles.

True False

2. Triangles are never classified by the properties of their sides.

True **False**

10.7 Classifications of Triangles by Sides

1. The classification of a triangle by sides is determined by the number of **congruent** sides

10.8 Equilateral Triangle

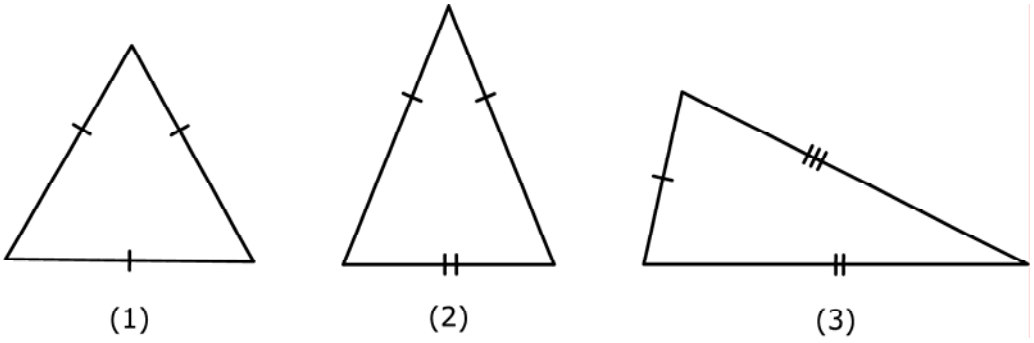
1. If a triangle has three congruent sides it is an **equilateral** triangle.

2. Triangle △ABC has sides such that AB = 7, BC = 7, CA = 7. Describe this triangle.

Because all three sides are of the same length, it is an equilateral triangle.

3. Are all triangles equilateral triangles. **No.**

4. Which triangle is an equilateral triangle?



Triangle number (1)

10.9 Isosceles Triangle

1. An isosceles triangle has **at least two** sides that are the **same length.**

2. An equilateral triangle is an isosceles triangle.

True False (It has at least two sides the same length. In fact, it has three sides the same length)

3. Which triangle in the drawing above is an isosceles triangle?

triangle number (2)

10.10 Scalene Triangle

1. A scalene triangle has sides that are **all of different length**.

2. Which triangle in the drawing above is a scalene triangle?

triangle number (3)

10.11 Summary of Classification of Triangles by Sides

1. How many congruent sides does a scalene triangle have?

A. One.
B. Two.
C. Three.
D. None of the above.

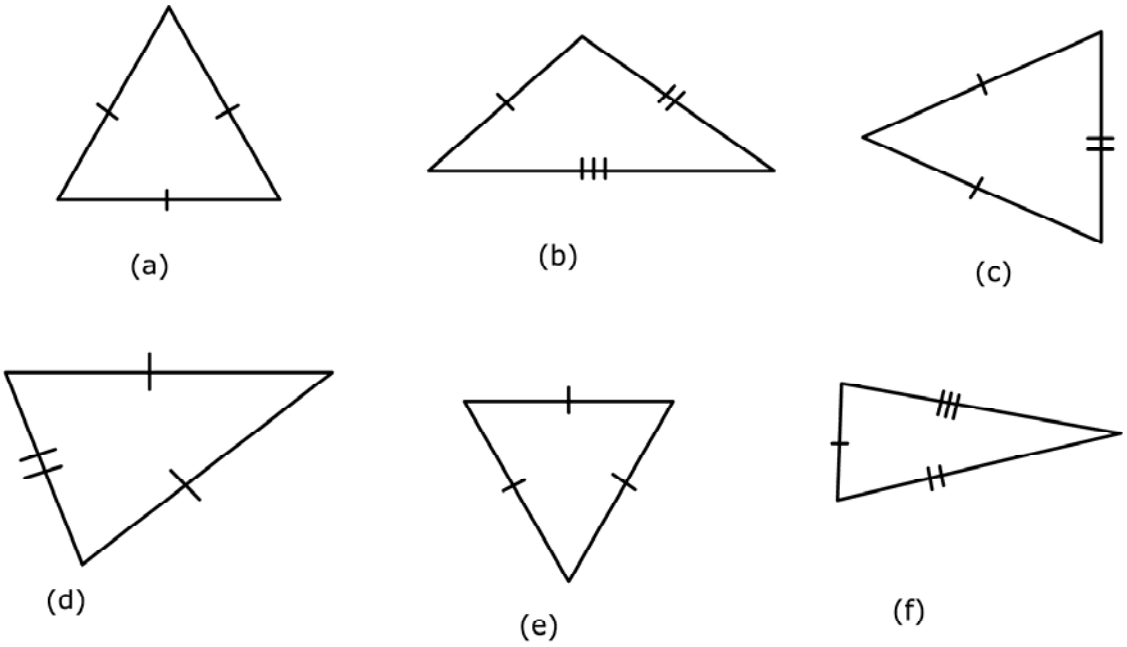
2. How many congruent sides does an isosceles triangle have?

   A. One.
   B. Two.
   C. Three.
   D. None of the above.

3. How many congruent sides does an equilateral triangle have?

   A. One.
   B. Two.
   C. Three.
   D. None of the above.

4. In the drawing, identify the equilateral triangles, isosceles triangles, scalene triangles.



Equilateral a, e. Isosceles c, d. Scalene b, f.

10.12 Classification of Triangles by Angles

1. Triangles can be classified by angle.

   True False

2. Triangles are classified by whether or not they contain an angle less than, equal to, or greater than:

   A. 30°
   B. 60°

C. 90°
D. 180°

3. Triangles are classified by whether or not they have congruent angles.

True False

10.13 Acute Triangles

1. Triangles that have interior angles that are all less than 90° are called **acute** triangles

2. An acute angle is

A. greater than 90°.
B. equal to 90°.
C. less than 90°.
D. none of the above.

3. Triangle △ABC has two interior angles of 40° and 50°. Is it an acute triangle?

No. The third angle is 90°. Acute Angles are less than 90°.

4. Triangle △JKL has two interior angles of 50°. What type of triangle is it?

Acute triangle. The third angle is 80°

5. Given triangle △TJY, if $m\angle T = 80°$ what must be true of ∠J and ∠Y for this triangle to be an acute triangle?

Both angles must be less than 50°.

10.14 Right Triangles

1. Triangles that have an interior angle equal to 90° are called **right triangles**.

2. A right angle is:

A. greater than 90°.
B. equal to 90°.
C. less than 90°.
D. none of the above.

3. Given △WDR, if the exterior angle at vertex W is 90°, what type of triangle is this?

Right triangle.

4. Triangle △ABC has two interior angles of 45°. Is this a right triangle?

Yes. The third angle is 90°.

5. In what case can an interior angle be equal in measure to an exterior angle?

If the measure is equal to 90°. That is, if the triangle is a right triangle.

6. Can a triangle contain two right angles?

No. The sum of the interior angles is 180°. You can't have two 90° angles.

10.15 Obtuse Triangles

1. Triangles that contain an angle greater than 90° are called **obtuse triangles**.

2. An obtuse angle is

A. greater than 90°.
B. equal to 90°.
C. less than 90°.
D. none of the above.

3. Triangle $\triangle DRF$ has two interior angles equal to 40°. What type of triangle is $\triangle DRF$?

An obtuse triangle. The third angle is 100°.

4. Triangle $\triangle ABC$ has an exterior angle equal to 50°. What type of triangle is it?

An obtuse triangle. The interior angle is 130°, an obtuse angle.

5. Can an obtuse triangle also be a right triangle?

No, the sum of the interior angles of the triangle is 180°. Only one angle may be equal to or greater than 90°.

6. If there is an obtuse angle in a triangle, then the other two angles *must* be acute.

True False

10.16 Equiangular Triangles

1. The triangle that contains three angles of equal measure is called an **equiangular** triangle.

2. Equilateral triangles are equiangular triangles.

True False

3. All equiangular triangles have exactly the same interior angle measure.

True False

4. Triangle $\triangle QRS$ has an exterior angle equal to 120°, is it **possible** that this triangle is an equiangular triangle? Is it guaranteed?

Yes, it is possible. The interior angle is 60°. No, it is not guaranteed. The other two interior angles would also need to be 60°.

10.17 Summary of the Triangle Family

1. Triangle $\triangle ABC$ contains a 90° angle. What type of triangle is it?

A right triangle

2. Triangle $\triangle MNB$ has three sides that are all different lengths. What type of triangle is it?

   A scalene triangle

3. Is there such a thing as an equilateral right triangle?

   No. The interior angles of an equilateral triangle are all 60°.

4. A given triangle has an interior angle of 100°, what type of triangle isit?

   An obtuse triangle

5. What is the maximum number of right angles in a triangle?

   A. zero
   B. one
   C. two
   D. three

6. What is the maximum number of obtuse angles in a triangle?

   A. zero
   B. one
   C. two
   D. three

7. Can a right triangle also be an isosceles triangle? If so, what are the measures of its interior angles?

   Yes. 90°, 45°, 45°.

8. Are all equilateral triangles equiangular?

   Yes. All interior angles of an equilateral triangle are 60°.

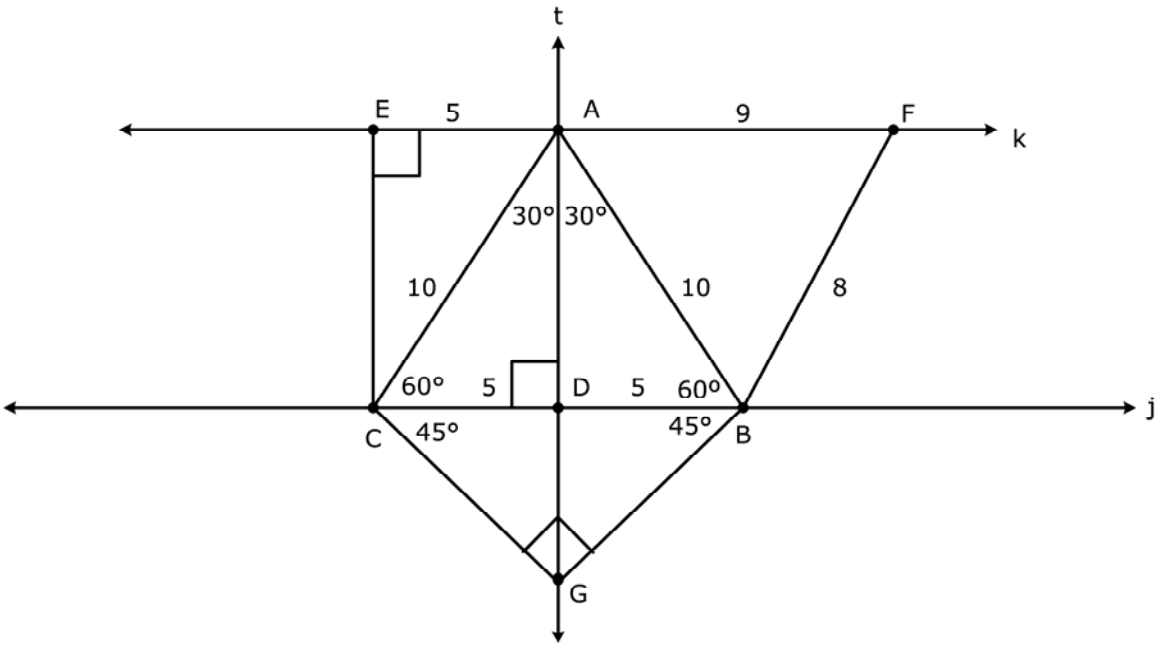
9. Are all equiangular triangles are equilateral?

   Yes. We will prove this in a later chapter.

10. A given triangle has two sides of equal length. What type of triangle is it?

    An isosceles triangle.

For questions 11-20, use the drawing below:



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11. What type of triangle is △ABF? **Scalene triangle.**

12. What type of triangle is △ABC? **Equilateral triangle. Equiangular triangle.**

13. What is $m\angle ECA$? **30°**

14. Is △CGB a right triangle? **Yes.**

15. Is △ACD ≅ △ABD? **Yes.**

16. What type of triangle is △ABG? **An obtuse triangle.**

17. What is $m\angle EAC$? **60°. Alternate interior angle of ∠DCA.**

18. Is $m\angle DGB = m\angle GCB$? **Yes. They are both 45°.**

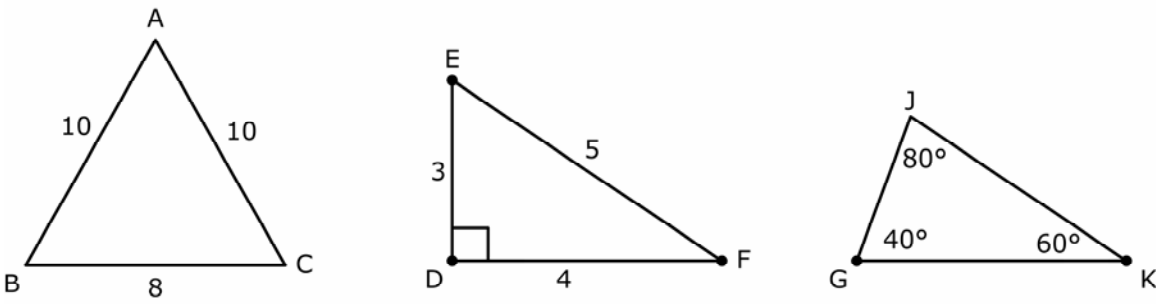
19. If $m\angle ABF = 80^\circ$. What is $m\angle AFB$? **40°**

20. If $m\angle AFB$ equals 40° and $m\angle ABF = 80^\circ$, what type of triangle is △AFB? **Acute triangle.**

21. Is t⊥k? **Yes.**

10.18 Names for the Sides and Angles of Triangles

For questions 1-15, use the drawing below:



1. In an isosceles triangle the two sides of equal length are called **legs**.

2. In △ABC what is $\overline{AC}$ called? **Leg**.

3. In △ABC what is $\overline{BC}$ called? **Leg.**

4. In △ABC what is ∠BAC called? **Vertex angle.**

5. What is ∠ABC? **Base angle.**

6. What is ∠ACB? **Base angle.**

7. What is $\overline{ED}$ called? **Leg.**

8. What is $\overline{DF}$ called? **Leg**

9. What is $\overline{EF}$ called? **Hypotenuse.**

10. If $m\angle ABC = 70^\circ$, what is $m\angle ACB$? **70°.**

11. What kind of angle is ∠EDF? **Right angle.**

12. Is $AB \cong AC$? **Yes.**

13. Is △JKG a scalene triangle? **Yes. We will prove this in the next few chapters**.

14. What is $\overline{JG}$ called? **Side.**

15. Does $m\angle DEF = m\angle EFD$? **No. The sides have different lengths.**

10.19 Medians, Angle Bisectors, and Altitudes of Triangles

1. All line segments in a triangle have the same name.

True **False**

10.20 Bases of Triangles

1. Any side of a triangle can be called its **base**.

2. How many base(s) does a scalene triangle have?

   A. 0
   B. 1
   C. 2
   D. 3 All triangles have three bases.

3. How many base(s) does an isosceles triangle have?

   A. 0
   B. 1
   C. 2
   D. 3 All triangles have three bases.

4. How many base(s) does a equilateral triangle have?

   A. 0
   B. 1
   C. 2
   D. 3 All triangles have three bases.

10.21 Altitudes of Triangles

1. The altitude of a triangle is the shortest line segment, starting from the line which contains the base of the triangle and ending at the vertex point opposite that base.

   True False

2. An altitude is always perpendicular to the base.

   True False

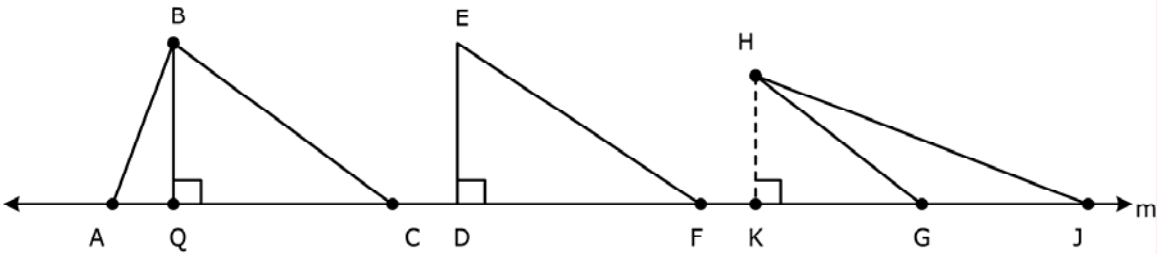
3. An altitude is always inside the triangle.

   True **False**

4. Where is the altitude on an obtuse triangle?

   Outside the triangle.

For questions 5-11, use the drawing below:



5. Which triangle is an acute triangle? **△ABC**

6. Which triangle is a right triangle? **△DEF**

7. Which triangle is an obtuse triangle? **△GHJ**

8. What is the altitude of the acute triangle? **$\overline{QB}$**

9. What is the altitude of the obtuse triangle? **$\overline{HK}$**

10. What is the altitude of the right triangle? **$\overline{ED}$**

11. At what angle do these altitudes intersect line m? **90°**

10.22 Intersection of Altitudes in Triangles

1. How do the three altitudes in the triangle intersect? **At a single point.**

2. In an acute triangle, where do the altitudes intersect? **Inside the triangle.**

3. In a right triangle, where do the altitudes intersect? **At the vertex of the right angle.**

4. In an obtuse triangle, where do the altitudes intersect? **Outside the triangle**.

5. We are given a triangle, △TXD. The altitude of the triangle is $\overline{XD}$. What do we know about this triangle?

It is a right triangle since the altitude is also leg.

6. We are given a triangle, △MNG. If the altitudes of this triangle intersect outside the triangle, what do we know about the largest angle ∠M?

It is larger than 90°. It is an obtuse triangle because the altitudes intersect outside.

7. In a given triangle, the altitude divides the vertex angle into two equal angles. What kind of triangle is this?

An isosceles triangle.

10.23 Angle Bisectors of Vertex Angles of Triangles

1. An **angle bisector** divides an angle into two angles of equal measure.

2. All triangles have **three** angle bisectors.

3. The three angle bisectors intersect at a **single point**.

4. A 40° angle is bisected into what? **Two 20° angles.**

5. What do we know about a triangle whose bisected angles are all congruent?

   It's an equilateral triangle.

10.24 Medians of Triangles

1. A median of a triangle is a line segment with one endpoint on the **vertex** of an angle and the other endpoint on the **midpoint** of the side opposite that angle.

2. The three medians of a triangle:

   A. intersect at a single point.
   B. intersect inside the triangle.
   C. have their endpoint at the midpoint of a side.
   D. none of the above.
   E. all of the above.

3. The three medians of an obtuse triangle intersect outside of the triangle.

   True **False**

4. If BD = CD, what do we know about AD? **It is a median**